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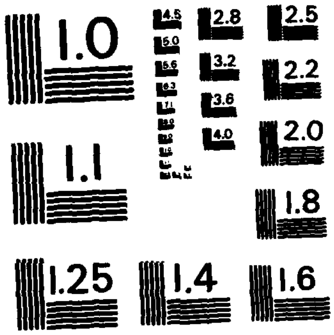

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Full Pseudospectral Solution to the Euler Equations of Motion for Airfoil Flow at Transonic Speeds

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NAVAL RESEARCH LABORATORY
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The unsteady Euler equations of motion are solved for by full pseudospectral computational techniques. Flows about a five percent half thick biconvex airfoil are obtained. Subcritical and supercritical cases are treated.

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CONTENTS

Full Pseudospectral Solution to the Euler Equations of Motion for Airfoil Flow at Transonic Speeds

I. INTRODUCTION

Pseudospectral solutions to one-dimensional compressible flow problems with discontinuities were first obtained by Gottlieb (reference 1) and Taylor (reference 2). Such flows are governed by the Euler equations. The present author has dealt with many such problems (references 3 - 6) ranging from the simple propagation of a normal shock wave to more complex, multiple shock wave flows. Solutions to bursting diaphragm flows, where an expansion fan, contact surface and shock front simultaneously exist and propagate, were obtained in reference 6. In addition, the time-dependent flow which arises from the collision of two normal shock waves (not necessarily equal strengths) was also obtained. The present author's work has demonstrated conclusively that artificial viscosity together with spectral filtering make pseudospectral techniques well suited to the solution of such compressible flow problems.

The first extension of pseudospectral techniques to two-dimensional, inviscid flows took place in 1982 when the present author succeeded in obtaining a numerical solution to the full Euler equations for supersonic wedge flows, (reference 3). The present work further extends the range of applicability of pseudospectral techniques to two-dimensional inviscid flows. Two-dimensional transonic airfoil solutions to the full Euler equations of motion have been obtained by full pseudospectral means (i.e. spatial derivatives in all directions treated spectrally). Solutions including embedded supersonic zones and discontinuities will be presented. They represent the subcritical, supercritical, and supersonic flow regimes encountered in the full transonic domain.

Manuscript approved August 1, 1985.

II. Pseudospectral Methods

Pseudospectral solution techniques involve the use of series of functions to represent the global properties of a flow field and its spatial derivatives. In the present work Chebyshev polynomials are used. They are reprensented by $T_n(x)$ where

$$T_n(x) = \cos\left[n \cos^{-1}(x)\right] , \tag{1a}$$

or

$$T_n(\Theta) = \cos\left[n\,\Theta\right] ; \tag{1b}$$

$$\Theta = \cos^{-1}(x) . \tag{1c}$$

A function $F(x,t)$ may be represented as

$$F(x,t) = \sum_{n=0}^{N} A_n(t)\, T_n(x) ; \tag{2}$$

where the time dependence is contained in the spectral coefficients, $A_n(t)$; and the spatial dependence is in the Chebyshev polynominals.

The two-dimensional, time-dependent Euler equations of motion, cast in conservation law form, are given by

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{E}}{\partial x} + \frac{\partial \vec{F}}{\partial y} = 0 ; \tag{3a}$$

where

$$\vec{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix} \qquad \vec{E} = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ \rho uv \\ (e + p)u \end{bmatrix} \tag{3b}$$

$$\vec{F} = \begin{bmatrix} \rho v \\ \rho uv \\ p + \rho v^2 \\ (e + p)v \end{bmatrix}$$

$$e = \frac{p}{\gamma - 1} + \frac{\rho(u^2 + v^2)}{2} .$$

These equations are solved using Chebyshev series to obtain the spatial derivatives and finite differences to obtain the temporal derivative. The Adams-Bashforth scheme was used in the present work for the evaluation of the time derivatives.

The Chebyshev polynominals are applied at discrete values of x and y, called collocation points x_j, y_i. The values are given by

$$X_j = \cos\left[\frac{\Pi j}{N_x}\right] \qquad 0 \leq j \leq N_x ; \tag{4a}$$

$$Y_i = \cos\left[\frac{\Pi i}{N_y}\right] \qquad 0 \leq i \leq N_y ; \tag{4b}$$

where N_x and N_y are the total number of modes used to represent the functions $\vec{E}(x,t)$ and $\vec{F}(y,t)$ given in equation (3b) The functions $\vec{E}$ and $\vec{F}$ are represented pseudospectrally by

$$\vec{E}\left(x_j,t\right) = \sum_{n=0}^{N_x} A_n(t)\, T_n\,(x_j) ; \tag{5a}$$

$$\vec{F}\,(y_i,t) = \sum_{n=0}^{N_y} B_n\,(t)\, T_n\,(y_i) . \tag{5b}$$

The spatial derivatives are given by

$$\frac{\partial\vec{E}}{\partial x_j} = \sum_{n=0}^{N_x} A_n(t)^{(1)}\; T_n\,(x_j) ; \tag{6a}$$

$$\frac{\partial\vec{F}}{\partial y_i} = \sum_{n=0}^{N_y} B_n(t)^{(1)}\; T_n\,(y_i) ; \tag{6b}$$

where

$$A_n^{(1)}\,(t) = \frac{2}{C_n} \sum_{\substack{p=n+1\\ p+n=odd}}^{N_x} pA_p\,(t) ; \tag{7a}$$

$$B_n^{(1)}\,(t) = \frac{2}{C_n} \sum_{\substack{p=n+1\\ p+n=odd}}^{N_y} p\; B_p\,(t) ; \tag{7b}$$

In these expressions

$$C_o = 2 ; \tag{7c}$$

$$C_n = 1 , \quad n > 0 .$$

The A_n's and B_n's are determined using equations 5a and b. Inverse FFT operations are used to obtain the A_n's and B_n's from the known values of $\vec{E}$ and $\vec{F}$ at the current time step. Once the $A_n^{(1)}$ and $B_n^{(1)}$ values are obtained from the recurrence relations 7a and b, direct FFT operations are used to evaluate the sums in 6a and b to obtain the spatial derivatives at the current time step. Finally, the solution is advanced in time using the Adams-Bashforth algorithm:

$$\vec{U}^{\,t+\Delta t} = \vec{U}^{\,t} + \frac{3}{2} \Delta t \left[\frac{\partial \vec{E}}{\partial x} \right]^{t} - 1/2 \, \Delta t \left[\frac{\partial \vec{E}}{\partial x} \right]^{t-\Delta t}$$

$$+ \frac{3}{2} \Delta t \left[\frac{\partial \vec{F}}{\partial Y} \right]^{t} - 1/2 \Delta t \left[\frac{\partial \vec{F}}{\partial Y} \right]^{t-\Delta t} . \tag{8}$$

The low pass spectral filter developed by Gottlieb (reference 1) was used to damp spectral oscillations. It is given by

$$e^{-\alpha \left[\frac{K - Ko}{K_{max} - K_o} \right]^4} ; \tag{9}$$

where K is the spectral wavenumber and K_{max} is the maximum wavenumber corresponding to the total number of collocation points. The term K_o is given by $\frac{5}{6} K_{max}$. In addition to this spectral filter, 2nd order artificial viscosity was globally applied in both the x and y directions. Its form is given by

$$D_{n,i,j} = - \mu_x \left[U_{n,i,j+1} - 2U_{n,i,j} + U_{n,i,\, j-1} \right] \tag{10}$$

$$- \mu_y \left[U_{n,i+1,j} - 2 \, U_{n,i,j} + U_{n,i-1,j} \right] .$$

This term is applied to the right hand side of 3a. In equation 10, μ_x and μ_y are the magnitudes of the artificial viscosity coefficients.

III. Results

The airfoil geometry used was a five percent half thickness ratio biconvex shape. Free stream Mach numbers of 0.70, 0.84 and 1.05 were run. The first yields subcritical flow over the airfoil surface. The second yields an embedded supercritical flow zone at subsonic free stream and the third also subcritical but at supersonic free stream conditions. For all cases 128 points were used in the x-direction and 32 points in the y-direction. Characteristic boundary conditions were used at subsonic inflow and outflow boundaries. Flow variables were held fixed at supersonic inflow boundaries and allowed to float at supersonic outflow boundaries. Surface tangency was applied at the upper and lower (airfoil) computational boundaries.

Several coordinate transformations were applied to generate an appropriate distribution of points in the flow field. The final computational coordinates are obtained by the following sequence:

$$(x,y) \rightarrow (\xi,\eta) \rightarrow (\bar{\xi},\zeta) \rightarrow (\bar{\xi},\bar{\zeta}) \tag{11}$$

where

$$x = A\,\xi\,\frac{1-C_1\alpha\xi^2}{(1-\xi^2)^\alpha} - C_3\left[1-\frac{\xi^2-\xi^2_{te}}{\xi_1^2-\xi^2_{te}}\right]\xi$$

$$-x_{max} < x < x_{max} \qquad -\xi_1 < \xi < +\xi_2 \tag{12a}$$

$$\bar{\xi} = 2\left[\frac{\xi-\xi_1}{\xi_2-\xi_1}\right] - 1 \qquad -1 < \bar{\xi} < +1 \tag{12b}$$

and

$$y = A\eta\,\frac{1-C_1\,\alpha\,\eta^2}{(1-\eta^2)^\alpha} \qquad 0 < y < y_{max} \tag{13a}$$

$$\zeta = \frac{\eta-\eta_{min}(\bar{\xi})}{\eta_{max}-\eta_{min}(\bar{\xi})} \qquad 0 < \zeta < \zeta_{max} \tag{13b}$$

$$\bar{\zeta} = 2\,\zeta - 1 \qquad -1 < \bar{\zeta} < +1 \tag{13c}$$

The transformation given by the first term of equation 12a is the same as **that** used in reference 7. The second term of equation 12a is one developed by the present author. The new transformation represented by both terms of 12a clusters points about the airfoil leading and trailing edges. The x-coordinate in 12a is the collocation point coordinate (cosine clustered) and ranges from $-x_{max}$ to $+x_{max}$. The ξ coordinate is an intermediate coordinate which is clustered about the leading and trailing edges and ranges from $-\xi_1$, to $+\xi_2$ (presently $|\xi_1| = |\xi_2|$). Finally, $\bar{\xi}$ is introduced to obtain the required range of coordinate values for the pseudospectral computations, namely $-1 \leq \bar{\xi} \leq +1$.

The stretching function for y, equation 13a, takes y to η where, $0 \leq y \leq y_{max}$ and $0 \leq \eta \leq \eta_{max}$. The ζ transformation is introduced to make the airfoil surface a constant coordinate line. The range of ζ is $0 \leq \zeta \leq \zeta_{max}$. Again, $\bar{\zeta}$ is introduced to insure the proper range of coordinates namely, $-1 \leq \bar{\zeta} \leq +1$.

The subcritical case results are shown in figures 1 and 2 . Comparisons with the finite difference potential flow solution of reference 7 are shown in figure 3. Grid resolution for the potential flow solution was 90 X 21 (x,y). Two pseudospectral cases were run at this condition. Grid resolution was 64 X 16 and 128 X 32 respectively. (Potential flow data exists only on the airfoil surface, so the figures do not show any comparison off-surface.) The lower resolution run roughly approximates the potential flow solution. The higher resolution run is however a more accurate solution of the Euler equations. The mild oscillations aft of the airfoil trailing edge are not due to any instabilities. Rather they are dependent upon the magnitude of the artificial dissipation and can be eliminated simply by increasing the smoothing in the x-direction. Results which are presented here were obtained at the minimum values of smoothing in x and y. Pressure contours of the flow field are shown in figure 2. (For all results herein, the iterations were stopped when the airfoil surface flow was deemed converged. Therefore, full field convergence has not been reached and accounts for the non-uniformity of pressure contours interior to the field.) The contours are smooth, with no oscillatory behavior at all.

Results for the supercritical case at subsonic free stream are shown in figures 4 and 5 . Figure 5 shows comparison with a finite difference potential flow solution. The surface pressure coefficient distribution shows the presence of a shock wave at a value of X of about 0.20 for the Euler pseudospectral run versus 0.25 for the finite difference potential flow run. Of more importance is the fact that the sharpness of the shock is the same for both solutions. The pseudospectral technique resolves the shock extremely well. Pressure contours are shown in figure 6. The shock wave is clearly evident.

A supersonic free stream case was also run to see how the pseudospectral method would work (no comparison data is available at the conditions chosen). Results are shown in figures 7 and 8. The surface Cp distribution shows the presence of a shock wave very near the trailing edge (x=0.5). The field pressure contours vividly confirm this. Again all contours are smooth and without numerical oscillation.

IV. CONCLUSIONS

The present work has shown that full pseudospectral solutions to the unsteady, two-dimensional Euler equations of motion are obtainable for transonic airfoil flows. Subsonic and supersonic free stream conditons are handled equally well. Embedded discontinuities are properly resolved both as to sharpness and position.

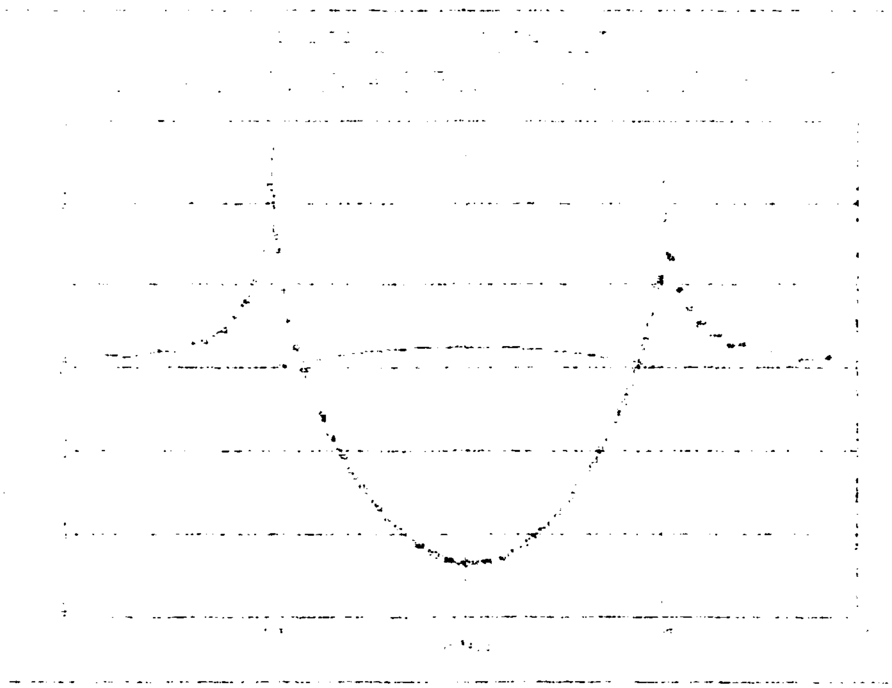

Fig. 1 — Surface pressure coefficient distribution for subcritical case

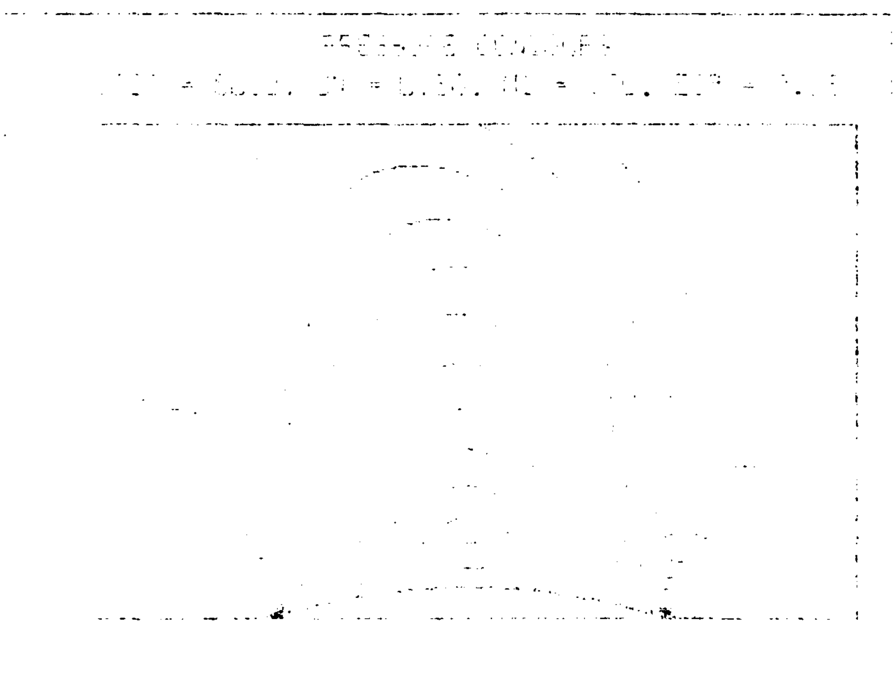

Fig. 2 — Pressure contours for subcritical case

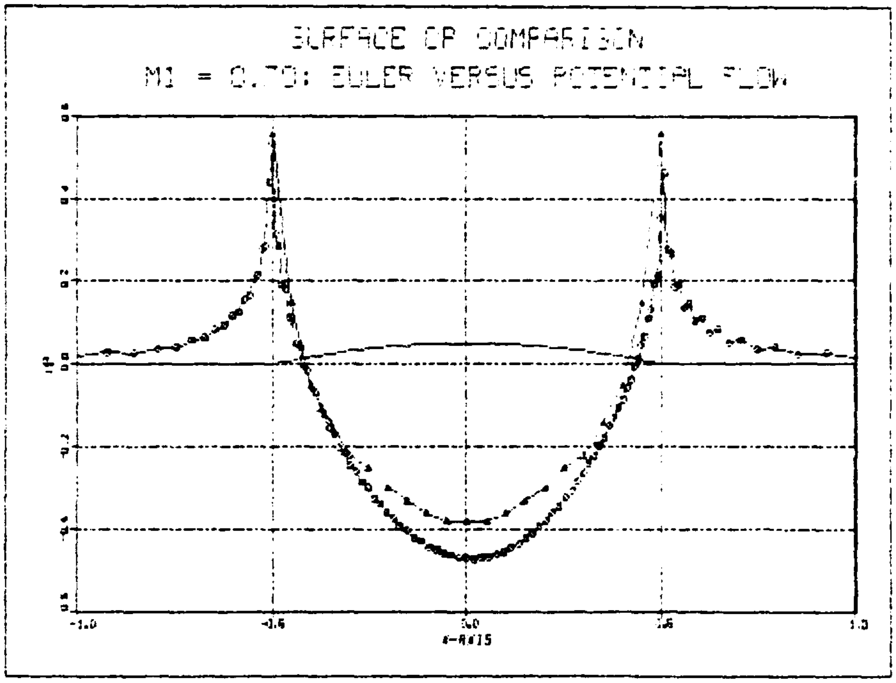

Fig. 3 — Comparison of pseudospectral Euler solution versus potential flow solution

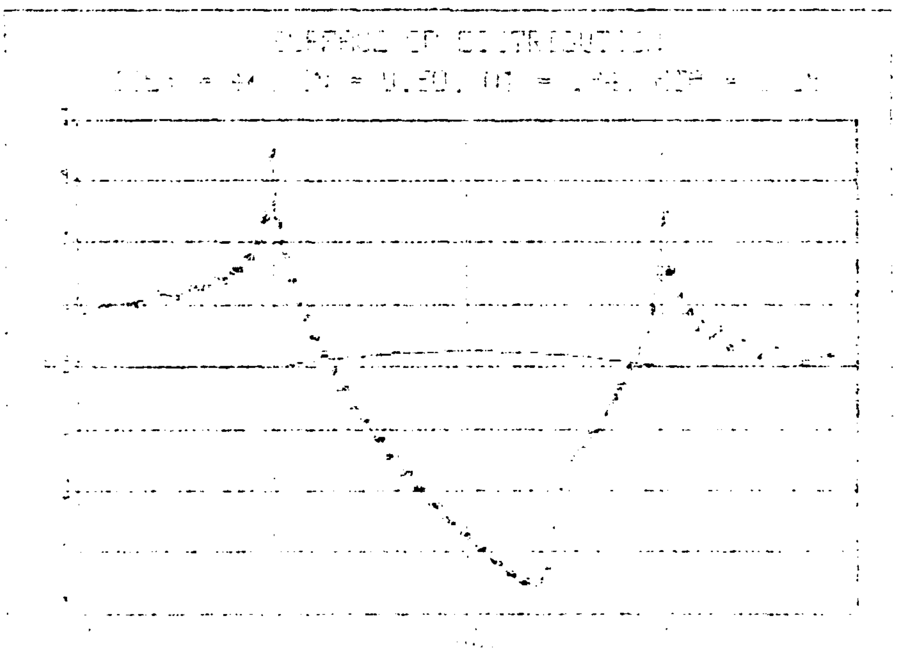

Fig. 4 — Surface pressure coefficient distribution for supercritical case

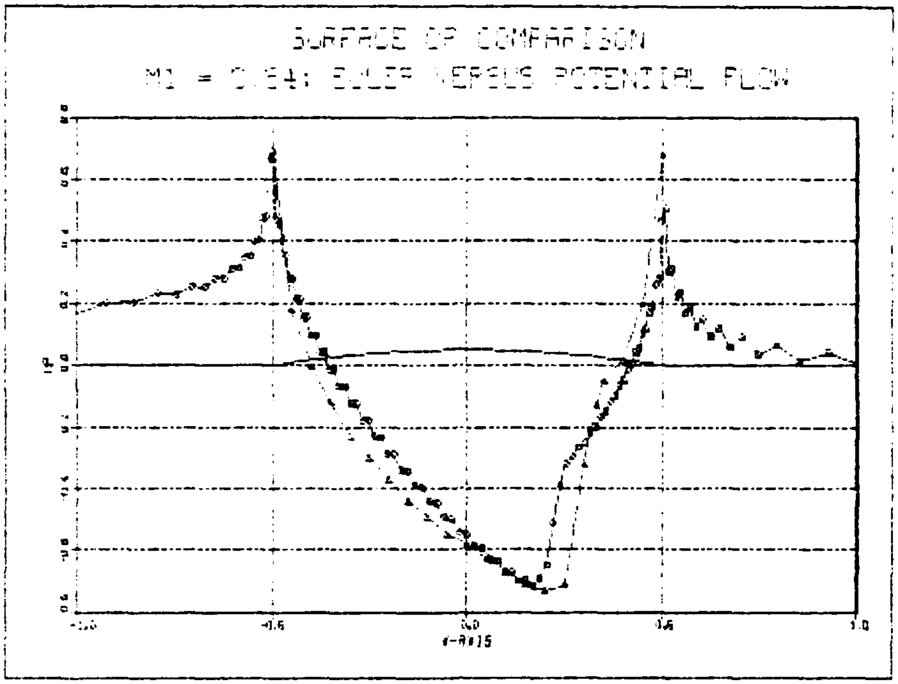

Fig. 5 — Pressure contours for supercritical case

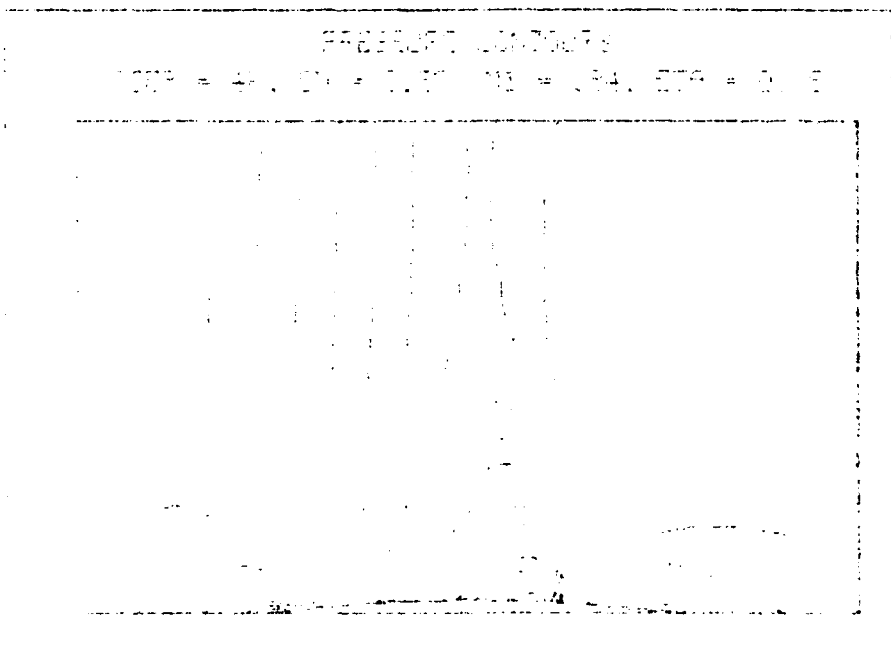

Fig. 6 — Comparison of pseudospectral Euler solution and potential flow solution

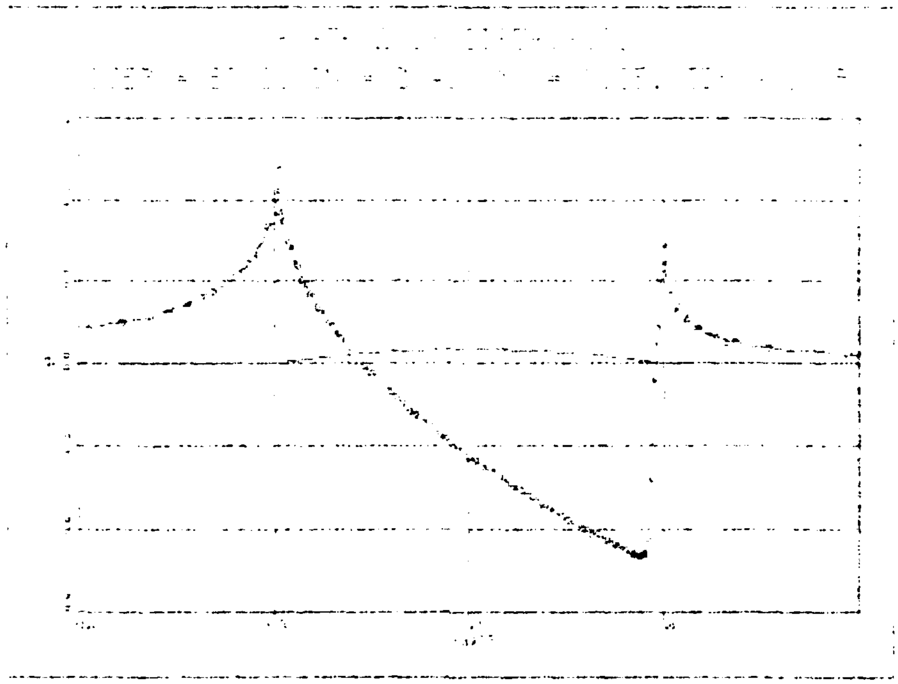

Fig. 7 — Surface pressure coefficient distribution for supersonic case

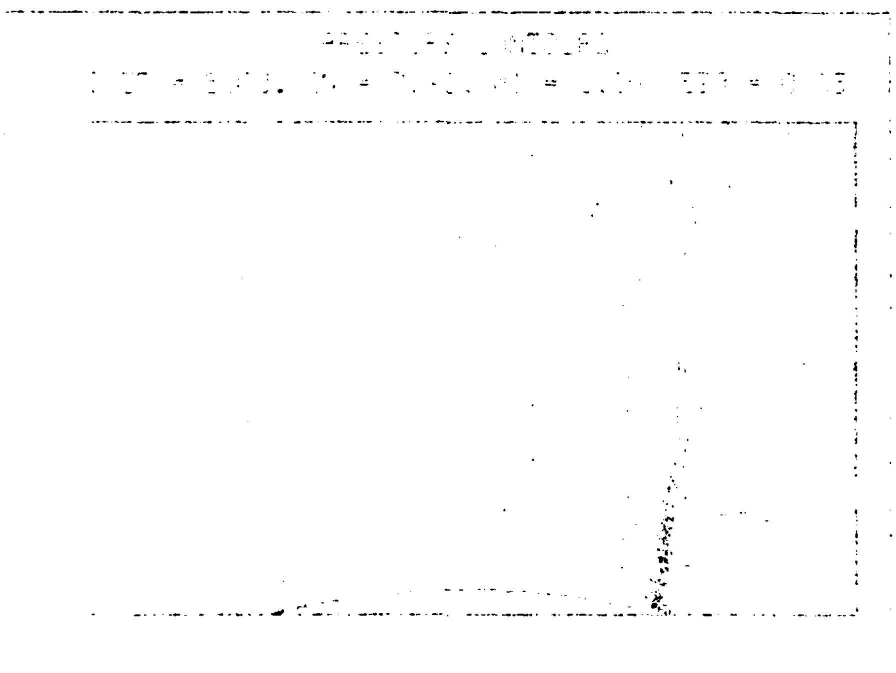

Fig. 8 — Pressure contours for supersonic case

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